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Reciprocity theorem of linear active two-port network

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***Abstract*—This paper proposes four methods for changing the linear active two-port networks into linear passive-like two-port networks. The reciprocity theorem is still valid in the linear passive-like two-port network, if the active linear network includes only independent sources. This paper extends the use of the reciprocity theorem into linear active two-port network.**

***Index Terms*—active two-port network, reciprocity theorem, passive-like network.**

I. Introduction

The reciprocity theorem can be derived from the Tellegen's Theorem [1].

The reciprocity theorem is applicable to linear passive two-port networks in three forms[2][3].

Form 1: The ratio of voltage measured across a pair of terminals to the excitation current applied at another pair of terminals is invariant to an interchange of excitation terminals and response terminals.

Form 2: The ratio of current measured in a short-circuit across a pair of terminals to the excitation voltage applied at another pair of terminals is invariant to an interchange of excitation terminals and response terminals.

Form 3: The ratio of current measured in a short-circuit across a first pair of terminals to the excitation current applied at the second pair of terminals is same as the ratio of voltage measured across the second pair of terminals to the voltage applied at the first pair of terminals

However, for linear active networks with independent source, the reciprocity theorem can not be used directly.

In this paper, we first change linear active networks with independent source into linear passive-like two-port networks. There are four forms passive-like networks, and reciprocity theorem is valid in linear passive-like two-port networks. Last we use reciprocity theorem in linear active networks.

II. Definition and Types of Passive-like Networks

By means of series connecting one voltage source or parallel connecting one current source to the each port of the linear active two-port network, we can get four kinds of balancing modes and therefore four kinds of balanced linear two-port networks are created.

The short-circuit current and open-circuit voltage are both 0 on the two port of the balanced networks. Which are the properties of the passive networks. So it is called "passive-like network".

The four kinds of balancing methods are described below

A. *Current-current balancing*

The balancing method is to measure the short-circuit current of the network's two ports. And connect a current source to the each port with the current value as the measurement result, the reference directions of the current source added are the same as the current measured before.

In this paper, port1 on the left side and port2 on the right side

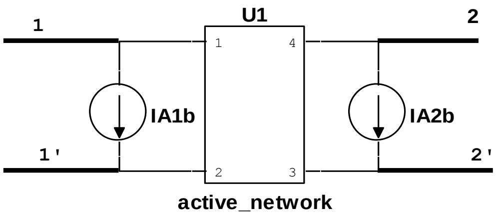

Figure 1 Passive-like network A from current-current balancing

IA1b/IA2b means passive-like network A Port1/2 balancing current.

B. *Voltage-voltage balancing*

The balancing method is to measure the open circuit voltage of the network's two ports. And connect a voltage source to the each port with the voltage value as the measurement result. Then the network is balanced. The reference directions of the voltage source added are the same as the voltage measured before.

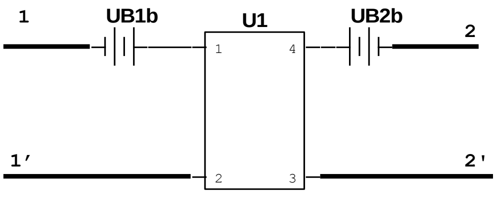

Figure 2 Passive-like network B from voltage-voltage balancing

UB1b/UB2b means passive-like network B Port1/2 balancing Voltage.

C. *Voltage-current balancing*

Firstly, measure the open circuit voltage of the network's port 1, and meanwhile measure the short-circuit current of the network's port 2. Secondly, connect a voltage source to the port 1 with the voltage value as the measurement result, and connect a current source to the port 2 with the current value as the measurement result

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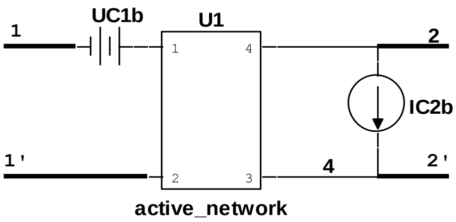

Figure 3 Passive-like network C from voltage-current balancing

UC1b means passive network C Port1 balancing Voltage. IC2b means passive network C Port2 balancing Current

D. *Current-voltage balancing*

Firstly, measure the short-circuit current of the network's port 1, and meanwhile measure the open circuit voltage of the network's port 2, Secondly connect a current source to the port 1 with the voltage value as the measurement result, and connect a voltage source to the port 2 with the voltage value as the measurement result.

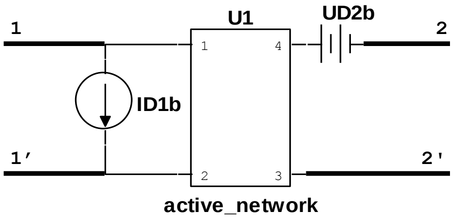

Figure 4 Passive-like network D from current–voltage balancing

ID1b means passive network D Port1 balancing Current. UD2b means passive network D Port2 balancing voltage.

III. PASSIVE-LIKE NETWORKS FROM LINEAR ACTIVE NETWORK WITH INDEPENDENT SOURCE

The Balance-source module in the figure 5 represents the voltage source or current source, as described in Chapter 2.

The Passive-like network can be divided into: passive network, current source and voltage source from original network, current source and voltage source form balancing.

The I_1、I_2、U_1，U_2 on the two port of the passive-like network are influenced by three part: Passive network, current source and voltage source from original network, current source and voltage source form balancing:

When the current source and voltage source from original network, current source and voltage source form balancing work together, they will be mutual eliminated base on the descriptor from Chapter 1.

$$I_1 = I_{1-passive} + I_{1-independent\ -source} + I_{1-balance\ -source} = I_{1-passive}$$

$$I_2 = I_{2-passive} + I_{2-independent\ -source} + I_{2-balance\ -source} = I_{2-passive}$$

$$U_1 = U_{2-passive} + U_{1-independent\ -source} + U_{1-balance\ -source} = U_{2-passive}$$

$$U_2 = U_{2-passive} + U_{2-independent\ -source} + U_{2-balance\ -source} = U_{2-passive}$$

Since the passive network is same and $I_{1-passive}$、$I_{2-passive}$、$U_{2-passive}$、$U_{2-passive}$ all accord with the description of reciprocity theorem. So although these four kinds passive-like network with different topology,I_1、I_2、U_1、U_2 will accord with the description of reciprocity theorem.

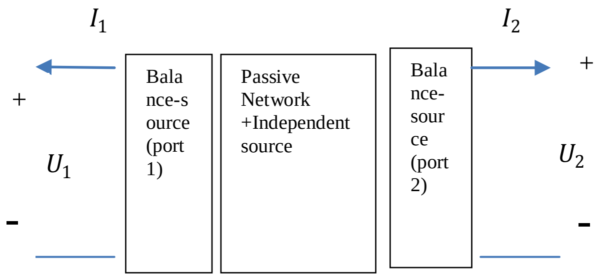

Figure 5 Four kinds of "Passive-like networks" by balancing

IV. RECIPROCITY THEOREM USED ON ACTIVE NETWORKS WITH INDEPENDENT SOURCE

There four kinds of passive-like networks balanced from the active network with only independent source. So we get six types of two-two combinations. And these six combinations all apply to the three special forms of the reciprocity theorem. We choose only three more important ones from those to descript.

A. *Current-current balancing vs. voltage-voltage balancing*

From an active network, we get passive-like network A Figure 6 and passive-like network B Figure 7 with port1 on the left side and port2 on the right side.

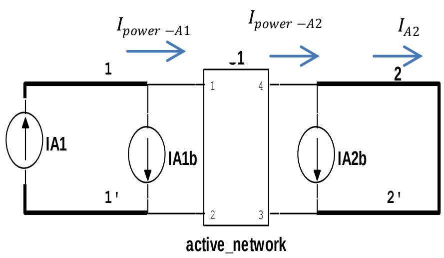

Figure 6 Passive-like network A Stimulated by I_{A1}

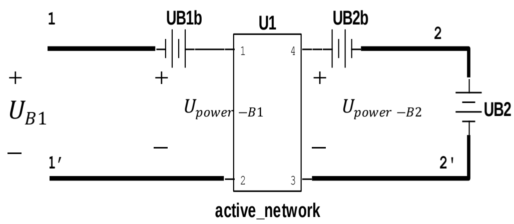

Figure 7 Passive-like network B Stimulated by U_{B2}

It is proved that the form 3 of the reciprocity theorem can be applied in the linear passive-like network.

$$\frac{I_{A2}}{I_{A1}} = \frac{U_{B1}}{U_{B2}} = K \quad (1)$$

For the active linear network with only independent source, it has:

$$I_{power\ -A1} + I_{A1b} = I_{A1}$$
$$I_{power\ -A2} - I_{A2b} = I_{A2}$$
$$U_{power\ -B2} - U_{B2b} = U_{B2}$$
$$U_{power\ -B1} - U_{B1b} = U_{B1}$$

Since I_{A1b}、I_{A2b}、U_{B2b}、U_{B1b} can be measured according to the description in Chapter 1, in addition to the formula 1, we can get the relationship of $I_{power\ -A1}$、$I_{power\ -A2}$、$U_{power\ -B2}$、$U_{power\ -B1}$ which used in active network.

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B. *Voltage-current balancing vs. current-voltage balancing*

From an active network, we get a linear passive-like network C Figure 8 and linear passive-like network B Figure 9 with Port1 on the left side and port2 on the right side.

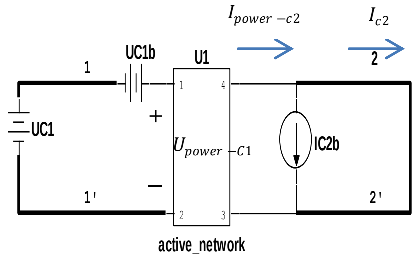

Figure 8 Passive-like network C Stimulated by U_{C1}

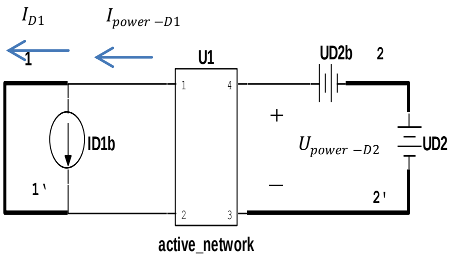

Figure 9 Passive-like network D Stimulated by U_{D2}

it is proved that the form 2 of the reciprocity theorem can be applied in the linear passive-like network.

If $U_{C1} = U_{D2}$, then $I_{C2} = I_{D1}$ (2)

For the active linear network with only independent source included, it has:

$$U_{power\ -C1} = U_{C1} + U_{C1b}$$
$$I_{power\ -C2} = I_{C2} + I_{C2b}$$
$$U_{power\ -D2} = U_{D2} + U_{D2b}$$
$$I_{power\ -D1} = I_{D1} + I_{D1b}$$

Since U_{C1b}、I_{C2b}、U_{D2b}、I_{D1b} can be measured, in addition to the (2), we can get the relationship of $U_{power\ -C1}$、$I_{power\ -C2}$、$U_{power\ -D2}$、$I_{power\ -D1}$ which used in active network.

C. *Current-voltage balancing vs. voltage-current balancing*

From an active network, we get a linear passive-like network D Figure 10 and linear passive-like network C Figure 11 with port1 on the left side and port2 on the right side.

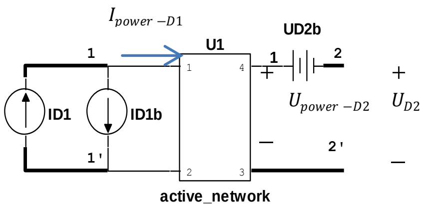

Figure 10 Passive-like network D stimulated by I_{D1}

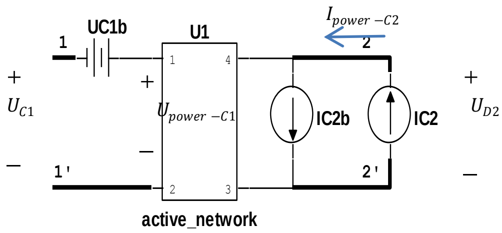

Figure.11 Passive-like network C stimulated by I_{C2}

It is proved that the form 1 of the reciprocity theorem can be applied in the linear passive-like network.

If $I_{D1} = I_{C2}$, then $U_{D2} = U_{C1}$ (3)

For the active linear network with only independent source included, it has:

$$I_{power\ -D1} = I_{D1} - I_{D1b}$$
$$U_{power\ -D2} = U_{D2} + U_{D2b}$$
$$I_{power\ -C2} = I_{C2} - I_{C2b}$$
$$U_{power\ -C1} = U_{C1} + U_{C1b}$$

Since I_{D1b}、U_{D2b}、I_{C2b}、U_{C2b} can be measured, in addition to the (3), we can get the relationship of $I_{power\ -D1}$、$U_{power\ -D2}$、$I_{power\ -C2}$、$U_{power\ -C1}$ which used in active network.

V. EXAMPLE VERIFICATION

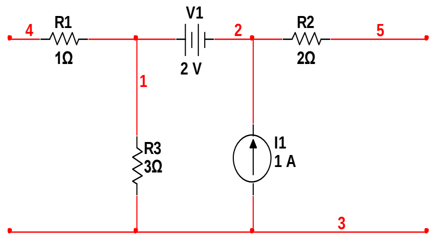

Figure .12 active network

Using active network shown in Figure 4.1 as an example, according to the description method in Chapter 1, the following table is obtained

Table.1 Balancing Method and Value

Balancing Method	Balancing Value
Current-Current Balancing	$I_{A1b} = 1.091A$ $I_{A2b} = -0.4545A$
voltage-voltage balancing	$U_{B1b} = 3V$ $U_{B2b} = 1V$
voltage-current balancing	$U_{C1b} = 2.4V$ $I_{C2b} = 0.2A$
Current-voltage balancing	$I_{D1b} = 0.75A$ $U_{D2b} = -1.25V$

A. *Current-current balancing vs. voltage-voltage balancing*

According to the description in chapter 3.1, the following table is obtained:

Table.2 Balancing Value in Current-current balancing vs. voltage-voltage balancing

$I_{power\ -A1} = 1A$	$I_{power\ -A1} + I_{A1b} = I_{A1} = 2.091A$
$I_{power\ -A2} = 0.8A$	$I_{power\ -A2} - I_{A2b} = I_{A2} = 1.2545A$
$U_{power\ -B2} = 2V$	$U_{power\ -B2} - U_{B2b} = U_{B2} = 1V$
$U_{power\ -B1} = 3.6V$	$U_{power\ -B1} - U_{B1b} = U_{B1} = 0.6V$

We can get:

$$\frac{I_{A2}}{I_{A1}} = \frac{U_{B1}}{U_{B2}} = K = 0.6$$

B. *Voltage-current balancing vs. current-voltage balancing*

According to the description in chapter 3.2, the following table is obtained:

Table3 Balancing Value in voltage -current balancing vs. current -voltage balancing

$U_{power\ -C1} = 8.4V$	$U_{power\ -C1} - U_{C1b} = U_{C1} = 6V$
$I_{power\ -C2} = 1.836A$	$I_{power\ -C2} - I_{C2b} = I_{C2} = 1.636A$

$U_{power\ -D2} = 4.75V$	$U_{power\ -D2} - U_{D2b} = U_{D2} = 6V$
$I_{power\ -D1} = 2.386A$	$I_{power\ -D1} - I_{D1b} = I_{D1} = 1.636A$

We can get:

$$U_{C1} = U_{D2} = 6V,\ \text{then } I_{C2} = I_{D1} = 1.636A$$

C. *Current-voltage balancing vs. voltage-current balancing*

According to the description in chapter 3.3, the following table is obtained:

Table.4 Balancing Value in current -voltage balancing vs. voltage-current balancing

$I_{power\ -D1} = 3A$	$I_{power\ -D1} + I_{D1b} = I_{D1} = 3.75A$
$U_{power\ -D2} = 10V$	$U_{power\ -D2} - U_{D2b} = U_{D2} = 11.25V$
$I_{power\ -C2} = 3.55A$	$I_{power\ -C2} + I_{C2b} = I_{C2} = 3.75A$
$U_{power\ -C1} = 13.65V$	$U_{power\ -C1} - U_{C1b} = U_{C1} = 11.25V$

We can get:

$$I_{D1} = I_{C2} = 3.75A,\ \text{then } U_{D2} = U_{C1} = 11.25V$$

VI. CONCLUSION

By means of series connecting one voltage source or parallel connecting one current source to the each port of the linear active two-port network, we can get four kinds of passive-like networks. These voltage source and current source are named balancing source in this paper. This paper descripts the method to get the balancing source.

If the active network includes only independent sources, then the four types of Passive-like networks conform to reciprocity theorem.

There four kinds of passive-like networks balanced from the active network with only independent source. So we get six types of two-two combinations. And these six combinations all apply to the three special forms of the reciprocity theorem. We choose only three more important ones from those to descript.

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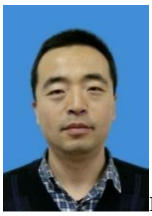

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